11.1 Videos Guide

11.1a

Exercise:

Calculate, to four decimal places, the first ten terms of the sequence and use them to plot the graph of the sequence by hand. Does the sequence appear to have a limit? If so, calculate it. If not, explain why.

- $a_n = 2 + \frac{(-1)^n}{n}$

Theorem (statement):

- If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.

Exercise:

- $a_n = 1 + \frac{10^n}{9^n}$

11.1b

Exercises:

Determine whether the sequence converges or diverges. If it converges, find the limit.

- $a_n = \frac{3+5n^2}{1+n}$
- $a_n = \frac{4^n}{1+9^n}$
- $a_n = e^{2n/(n+2)}$

Theorems (statement):

- If $\lim_{x\to\infty} f(x) = L$ and $f(n) = a_n$ when n is an integer, then $\lim_{x\to\infty} a_n = L$.
- If $\lim_{x\to\infty} a_n = L$ and the function f is continuous at L, then $\lim_{x\to\infty} f(a_n) = f(L)$.

11.1c

Exercises:

Determine whether the sequence converges or diverges. If it converges, find the limit.

- $\left\{\frac{\ln n}{\ln 2n}\right\}$
- $\frac{a_n(-3)^n}{n!}$

11.1d

Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

- $a_n = \frac{1-n}{2+n}$

Theorem (statement):

- Monotonic Sequence Theorem: Every bounded, monotonic sequence is convergent.

11.1e

Exercise:

- Show that the sequence defined by

$$a_1 = 2, \qquad a_{n+1} = \frac{1}{3-a_n}$$

satisfies $0 \leq a_n \leq 2$ and is decreasing. Deduce that the sequence is convergent and find its limit.